# Trajectories Control of a Projectile Using Genetic Algorithms in one Simulated Environment 

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#### Abstract

In this paper, we present an adaptive approach to the solution of an analytically solvable problem. We applied a Genetic Algorithm to find the best variation of a projectile's velocity orientation, in order to optimize the needed time and precision for one missile to find a mobile target. At the end of this work, the obtained results indicated a good performance of the G.A. compared to conventional methods.


## 1 ) Introduction

To solve a problem involving chaos without using chaos, even the fastest computers would require exponentially long programs or run times. However, a computer program can use chaos ( controlled randomness ) to rapidly generate solutions whose validity can be checked quickly, mimicking nature's technique for solving problems in evolution [Halliday 96]. Genetic Algorithms is an approach in evolutionary computing that handles the idea shown above.

This paper presents a proposal to solve a problem which solution is known and determinable by analytic methods, through the utilization of G.A. paradigm, with intent to produce parameters that will allow performance comparison between the two approaches. This proposal is justified by the large versatility that the application of G.A. has presented in the
solution of several kinds of problems, as seen in [Ribeiro 94], [Davis 91] and [Collins 96].

## 2 ) The Problem

The selected problem for the analysis was the control of the trajectory of a material point in a 3-dimensional space. This choice was based in the fact that the problem has an analytic solution and it is of easy understanding. In this work, we particularized the problem to the case in which a projectile must find a mobile target, that describes a pre-setted trajectory which is unknown by the projectile.

The problem was formalized in one differential equations system and simulated via numerical integration. At each iteration, the system recalculates the values of acceleration, velocity and position of the two material points and draws the actual state of the system in a 3dimensional space projected on the screen through a synthetic camera. The resulting force over the projectile is calculated taking the following components into account:

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Drag force exerted by the air: \(\mathrm{D}=1 / 2 \mathrm{C} \mathrm{\rho Av}^{2}\)
Weight force : \(\mathrm{F}_{\mathrm{W}}=\mathrm{m} . \mathrm{g}\)
Propulsion force: \(\mathrm{Fp}=\) constant
Buoyant Force : \(\mathrm{Fe}=\mathrm{g} \rho \mathrm{V}\)
    C Drag coefficient
    \(\rho \quad\) Air Density \(\left(1.21 \mathrm{~kg} / \mathrm{m}^{3}\right)\)
    A Area of the plain section perpendicular to the direction
        of the velocity vector
    v Module of the velocity vector
    m Mass of the projectile
    g Gravity acceleration \(\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\)
    V Volume of fluid displaced by the body
        All units are in the International System. .
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Fig. 1 - The involved forces
Assuming that the velocity of a projectile is commonly found in the range between two
and four times the speed of the sound, we made some simplifications considering its module constant, in order to improve the computational performance of the system. So, the resultant acceleration influences centripetally over the velocity, changing only its direction and not the module. The module's variation of $\theta$ and $\varphi$ angles cannot exceed the limit of $\pi / 16$ radians ( $\approx 11^{\circ}$ ).

## 3 ) Characteristics of the Developed System



Fig. 2 - Main screen
3.1 Genetic Representation of the Potential Solutions of the Problem

The binary number coding was chosen to represent the potential solutions of the proposed problem due to its properties of similarity to the biological chromosome, making easier the building of genetic operators. The size of the implemented chromosome was of sixteen bits, where the most and less significant bytes represent the respective angular changes in the components of the velocity in a determined interval $d t$, as shown in figure 3 .

The calculation presented in the following figure can be interpreted as a normalization of the A and B values, followed by a multiplication that represents the width of the variation band; and the subtractions that cuts the band in symmetric values between the limits of [ $-\pi / 16$ to $\pi / 16$ ].

Fig. 3 - Chromosome decoding
To generate the initial population, we set the state of all bits randomly, using an internal random number generator.

### 3.2 Fitness Function

The fitness of a chromosome was evaluated by calculating the variation of the component of the velocity vector over the axis that touches the two material points in space. To each chromosome, the G.A. calculates the fitness as the variation of the Euclidean distance between the projectile and the target, at the times $t$ and $t+d t$.


Fig. 4 - Fitness calculation

### 3.3 Genetic Operators

The implemented G.A. uses a variant of the classic one-point crossover. In this method, two points are selected randomly, one in each byte, where the cut, exchange and paste are made, recombining the chromosomes.

The angles $\theta$ and $\varphi$, which are represented respectively by the most and less significant bytes, remain independent between themselves, since the genetic operators transform each one individually.

The choose of parent chromosomes is made through the roulette wheel [Davis 91] method, where the probability of a chromosome to be selected is proportional to its fitness. Using the elitism method, the new generation will be formed by the individuals of best fitness of the previous generation and their direct offspring.

Over this new population of chromosomes is applied a single-bit mutation operator, where the selected bit is changed through a simple probabilistic test. For each bit inside a chromosome, one random number is generated inside the unitary interval and, if above the probabilistic rate of mutation, operates the alteration over the corresponding bit.

### 3.4 Used Parameters

During the implementation phase, some simple tests were made in order to find appropriate values to the parameters, restricting the covered interval to an optimal solution. To optimize the time/precision cost, the number of individuals at the population was set to thirtytwo individuals for each generation.

The number of needed generations for the projectile to find the target changes according with the parameters defined by the user. The maximum limit of generations was set to one thousand between the begin of the motion and the target hitting.

By the physical limitations of a highspeed projectile, we assumed that the acceleration vector would change the velocity in order to change only its orientation. The limit set to this variation was in the range between $-\pi / 16$ to $\pi / 16$.

## 5 ) Obtained Results

To validate the experiment, we built a set of graphics, obtained from a systematic variation in some involved parameters.


Fig. 5-Average iterations to hit the target versus the mutation percentile used

The graph of figure 5 was plot to evaluate the mutation percentile. Through the analysis of the results, we can perceive that the number of necessary iterations is smaller in the range among 0.1 and 0.3 .


Fig. 6 - Average iterations to hit the target versus missile velocity, using G.A.

Comparing the graphs 6 and 7, we conclude that there is not a significant detriment in the performance, in respect to number of iterations between G.A. and the conventional method.


Fig. 7 - Average iterations to hit the target versus missile velocity, using conventional methods.

## 6 ) Results Interpretation

Through the analysis of obtained results, we can infer:

- there is a optimal value for the mutation rate, just changing from problem to problem;
- the problem's solution are very sensible to the initial conditions, which improves the utilization of adaptive methods, like G.A.
- the complexity of the target trajectories has not an influence in the relative performance of the two methods. They have a similar performance for the same trajectory.


## 7 ) Future Improvements

7.1 Chromosome coding using real numbers

The innately variable involved in the problem ( continuous angles ), invokes the use of real numbers to represent the chromosome, in the way to prevent the process delay of coding and decoding needed by the binary representation. In order to do this, it is necessary to perform complete modification in the operators ( crossover and mutation ).

### 7.2 Cooperative and Co-Evolutive Approach

In a cooperative and co-evolutive approach, each variable is coded in an independent chromosome, evolving in different populations, in parallel. This approach could be
interesting in our model, which use two independence variables coded in the same chromosome.

## 8 ) Conclusions

At the end of the result analysis phase, we can observe that, although the Genetic Algorithm has a high computational cost, the quality of generated solutions is quite near of the obtained results by the conventional methods. To utilize an analytical method, we will usually demand a high knowledge and a rigid formalizing of the problem, plus a big precision and a high velocity to compute the results, which can not be always accepted. Based in this assertion, we can conclude that the application of Genetic Algorithm in problems that have an analytical solution is viable in a large variety of cases, and could be considered a generic method for optimization problems.

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